APPENDIX F

EXPERIMENTAL DESIGN STRATEGIES FOR DATA COLLECTION AND ANALYSIS

APPENDIX F. EXPERIMENTAL DESIGN STRATEGIES FOR DATA COLLECTION AND ANALYSIS

This appendix provides an introduction to data collection strategies and experimental design. These strategies are particularly useful for driver workload research where multiple factors must be included in an evaluation and orderly data collection is of great importance. Table F-1 provides definitions of terminology used in this appendix. Experimental design is discussed in many textbooks devoted to the subject (Keppel, 1991; Winer, 1971; Box, Hunter, and Hunter, 1978; Kirk, 1982).

FACTORIAL DESIGN APPROACHES

In discussing alternative data collection strategies and experimental designs, some specialized terminology will be used (see Table F-1). The most straightforward approach to experimental design is to use a factorial design. In this approach, all levels of all independent factors are crossed. For example, if there are 3 factors, each with two levels, there will be 2^3 or 8 combinations of the three factors in a full factorial experiment. The full factorial design is perhaps feasible with a relatively small number of factors (e.g., 2 to 4 factors). Full factorial experiments are appropriate when the evaluator requires that:

- Every main effect (i.e., effect of each factor averaged over all other factors) be estimated independently of every other factor, and
- all interactions be assessed.

It is important to note that for any experiment with n independent factors, there are 2^n effects that could be assessed. Thus for an experiment with 4 factors, there are 2^4 or 16 terms in a full regression model (4 main effects, 6 two-factor interactions, 4 three-factor interactions, 1 four-factor interaction, and the intercept). Generally speaking, higher-order (e.g., 3-factor and higher) interactions tend to be non-significant or account for a trivial proportion of the variability of the response measure (Box, Hunter, and Hunter, 1978). Furthermore, the interpretability of a higher-order interaction is usually extremely limited in supporting workload evaluations.

The selection of within-subjects experimental designs is generally beneficial from several standpoints. This type of design uses fewer subjects, as a rule, and reduces the variability within in each treatment level or treatment combination because subjects serve as their own controls. It is often the case that individual differences account for much larger proportions of variability in a measured response than any of equipment or roadway factors. However, repeated measures designs cannot be used if a) the number of treatment levels or treatment combinations is prohibitively larger, b) if there are potential practice effects (i.e., learning the experimental procedures, as apart from the in-vehicle technology), or c) if there are potential carry-over effects

Table 3-7. Experimental Design Terminology.

Experimental Design Term	**Definition**
Factor	An independent variable in the evaluation.
Factor Levels	The specific values of a factor included in the evaluation.
Factorial Design	An experimental design involving more than two or more factors, in which every combination of factor levels has been included. For example, if there are three factors (a, B, C) and each has two levels (a_1, a_2; b_1, b_2; c_1, c_2), there are 2^3 or 8 factorial combinations of the three factors: a_1, b_1, c_1 a_2, b_1, c_1 a_1, b_2, c_1 a_2, b_2, c_1 a_1, b_1, c_2 a_2, b_1, c_2 a_1, b_2, c_2 a_2, b_2, c_2
Treatment or Treatment Combination	a particular level (if only one factor is included) or combination of levels of factors (if two or more factors are included) in the evaluation.
Crossed Factors	Factors in which every level of one factor is combined with each level of another factor. In the example provided above, factors a, B, and C are crossed. Note that the number of treatments or treatment combinations can be calculated by multiplying the levels of each factor with the levels of the others. In the above example, each of three factors had the same number of levels and so there were 2 x 2 x 2 = 2^3 = 8 treatment combinations.
Nested factor	a factor B is said to be nested within one level of factor a if each level of factor B appears with only one level of factor a.
Main effect	The impact of a factor (independent variable) on a dependent variable independent of or averaged over the impact of all other independent variables.

Experimental Design Term	Definition
Interaction	An interaction is present when the effects of one factor depend on the levels of another factor. Only crossed factors may interact. For this reason, nested factors cannot be assessed for interaction with factor or factors within which they are nested.
Dependent Variable	This is the measured response variable analyzed to assess the effects of the factors included in the study. Also called Measure of Performance (MOP), criterion measure, and response measure.
Confounding (Aliasing)	A confound or alias is an effect that cannot be distinguished from another effect. Thus, confounds or aliases are confusions or uncertainties about the source of some effect. When considered formally in an experimental design, confounds and aliases refer to main effects or interactions in the model that cannot be distinguished statistically from other main effects or interactions in the model. If an extraneous or nuisance variable is not controlled properly, there can be a confound between that nuisance variable and some independent variable of interest.
Replication	The observation of two or more experimental units (e.g., subjects) under identical treatment conditions to obtain an estimate of experimental error or error variation and permit a more precise estimate of treatment effects.
Experimental Error (Residual)	Variation in a dependent variable that is attributable to factors not relevant to the research hypotheses, i.e., by random factors.
Between-subjects Design (Completely Randomized Design)	An experimental design in which each subject experiences only one treatment combination in the evaluation.
Within-subjects Design (Repeated Measures Design)	An experimental design in which each subject experiences all treatment combinations in the evaluation.
Mixed Design	An experimental design in which some factors in the evaluation are between-subject factors and other factors are within-subjects factors.
Random Factor	Factor for which the treatment levels are a random sample from a larger population and inferences will be drawn about this population.

Experimental Design Term	**Definition**
Fixed Factor	Independent factor for which all treatment levels about which statistical inferences are to be drawn are included in the study.

such as learning about the device or the roadway, noting that a simulator study includes crash hazard events, and so on. Mixed designs are useful for data economy and are mandatory when, for example, subject variables are a formal part of the study. For example, gender or age are between-subjects variables that might be included in a mixed design. Perhaps all subjects would be measured repeatedly under different driving conditions, in which case driving condition factors are within-subjects variables. Care must be taken in the choice of these experimental design alternatives and the choice of which approach to adopt depends on the availability of test subjects and the characteristics of the specific evaluation itself (Williges, 1984). Appendix F provides further descriptions of alternative experimental designs.

ECONOMIC DATA COLLECTION DESIGNS

Oftentimes, there will be constraints to the use of full factorial designs in device evaluations (Williges, 198 1). Some factors cannot be crossed in the real world. It-may not be possible to collect data from the entire factorial design at one time. There may be more factors than can be reasonably included in a full factorial experimental design. For these reasons, economical data collection approaches are a necessity and several different classes of approach will be presented here.

HIERARCHICAL DESIGNS

Hierarchical designs are suitable when it is appropriate to nest one or more factors into other factors. That is, levels of one factor appear only at one level of another factor in hierarchical fashion. The hierarchical design results in a smaller number of treatment combinations when compared to a complete factorial design. Because nested variables are not crossed with the factors they are nested within, it is not possible to assess any interactions that may be present. Thus, care must be taken in planning hierarchical designs when used for purposes of data collection economy (Williges, 1984).

CONFOUNDED FACTORIAL DESIGNS

Sometimes it is not possible to collect all the workload assessment data needed in a single data collection session. Thus, the evaluator must collect the data in stages or in “blocks”. There are procedures (e.g., Kirk, 1983) that can be used to define blocks and systematically confound or alias block differences with higher-order interactions. This allows for the assessment of all main effects and lower-order interactions at the expense of the higher order interaction used in developing the treatment combinations for the blocks. Tijerina et al. (in press) used a confounded factorial design to assess workload effects of lighting (night vs. day), traffic density (low vs. high) and road type (divided vs. undivided). It was not feasible to collect workload measures on a single driver in all 2^3 or 8 treatment combinations. Instead, it was only feasible to collect workload measures on four of the 8 treatment combinations. Figure F-l shows how the design was blocked across two groups of subjects such that the main effects and two-way interactions would not be confounded with possible differences across the two groups of subjects. Only the three-way interaction is completely confounded with the subject group effects. Since three-way and higher order interactions tend to be either not statistically significant or account for only a trivial proportion of the variability in a measure of performance, this was judged to be a worthwhile tradeoff.

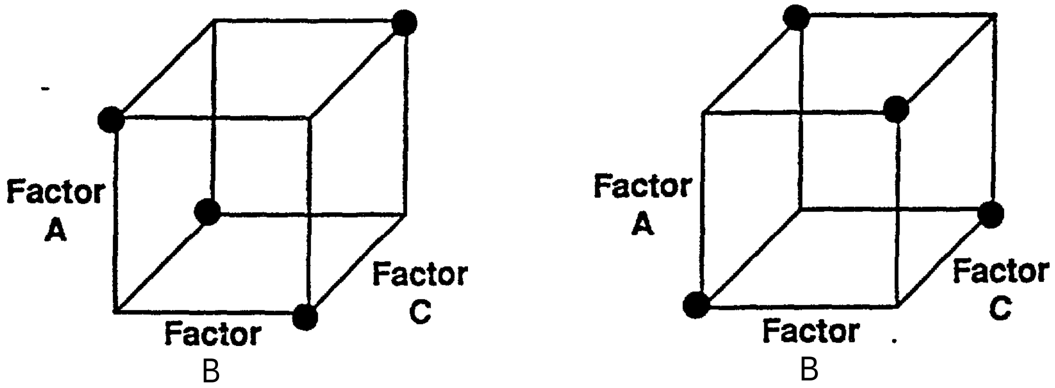
FRACTIONAL FACTORIAL DESIGNS

When it is impossible to collect data on all combinations of all factors or variables of interest from a higher-order design, a fractional factorial design may be used. As its name implies, a fractional factorial experimental design employs only a fraction of a full factorial design. That is, this experimental design approach and data collection procedure uses only a carefully chosen fraction of all possible combinations of factors to estimate the effects of those variables and interactions. For example, if there are eight (8) factors that might be included in an experiment or evaluation and each factor has only two levels, there are $2^8 = 256$ possible experimental treatments. On the other hand, a one-quarter fractional factorial design (2^{8-2}) requires only 64 experimental treatments to be run in an experiment. The data collection economy is bought with aliasing, i.e., confounding main effects and two-factor interactions with higher-order interactions. Higher-order interactions are assumed to be non-existent or trivial, So, if, for example, a two-factor interaction is statistically significant, it is attributed to that pair of factors rather than higher-order interactions aliased with it. The reasonableness of this approach comes from the fact that main effects tend to be larger than two-factor interactions, two-factor interactions tend to be larger than three-factor interactions, and so on (Box, Hunter, and Hunter, 1978). In addition to the general result that higher-order interactions contribute little or no additional explanatory power, the ability to comprehend and explain such interactions in a parsimonious way becomes impossible.

For most practical purposes, there is little advantage to evaluating the impact of factors alone and in two-factor interactions. This leads to the concept of design resolution. Design resolution refers to the precision of the estimated effects from the experiment and the types of aliases that might exist. In general:

- A design of Resolution R = III does not confound single-factor effects (called main effects) with one another but does confound main effects with two-factor interactions.

- A design of Resolution R = IV does not confound single-factor effects and two-factor interactions but does confound two-factor interactions with other two-factor interactions.



Conditions	Conditions
A_1, B_2, C_1	A_1, B_1, C_1
A_1, B_1, C_2	A_1, B_2, C_2
A_2, B_1, C_l	A_2, B_2, C_1
A_2, B_2, C_2	A_2, B_1, C_2

Figure F-l. Confounded Factorial Design Strategy for Assessing Driving Condition Factors and Their Effects on Driver Workload.

As mentioned previously, higher-order interactions (i.e., three-factor interactions and higher) are often not significant or account for so little of the variation in the response that they are trivial. For this reason, a significant main effect aliased with a higher-order interaction is attributed to the main effect alone. Similarly, a two-factor interaction that aliases with a three-factor interaction is attributed to the two-factor interaction alone. Since human performance and behavioral data sometimes yield two-factor interactions of interest, fractional factorial designs of resolution V or higher are recommended.

Originally, the fractional factorial approach to experimental design was developed to allow for sequential experimentation. That is, research or evaluations would be done in stages and ambiguities that arise in the first stage of the research could be investigated in subsequent data collection stages that dis-ambiguate the first stage results. It is often the case that product evaluations do not allow for sequential investigations, Thus, a resolution V design is recommended if only a single investigation is to be carried out. Finally, it should be noted that fractional factorial designs typically involve all factors at the same number of levels, most commonly 2 levels. Thus, the factors may be continuous variables with levels chosen for "high" and "low" or they may be dichotomous variables (e.g., male or female drivers).

The attachment to this appendix contains examples for Resolution III, IV, V, VI, and VII designs to study up to eight factors and indicates the number of unique runs required. The negative (-) sign and positive (+) signs represent the two levels of each factor. Assignment of factors and factor levels to codes is arbitrary. A software program has been developed, entitled the Automated Experimental Design (AED) Assistant, that will automatically generate fractional factorial design assignments for 2 level designs with up to 20 factors and a maximum of 256 treatment combinations, for 3-level designs with up to **12** factors and maximum of 243 treatment combinations, and for 5-level designs with up to 8 factors and a maximum of 125 treatment combinations (System Development Corporation, 1986).

CENTRAL COMPOSITE DESIGN APPROACHES

The fractional factorial design (along with confounded factorial designs and single-observation factorial designs) supports economical data collection for hypothesis testing, i.e., to test whether a driver-performance measure reliably varies with some workload factor. It is well suited to the analysis of categorical variables (at two levels, usually, though 3-level and 5-level fractional factorial designs are also possible). In other situations, however, the evaluator may seek to determine a quantitative relationship between driver performance and several quantitative independent factors, e.g., in-cab device parameters. Such a functional relationship is useful in that it allows for comparative predictions of various alternative system configurations, during, say, product development (Williges, 198 1). The empirical model form that has been recommended for human factors use is a second-order polynomial, i.e.,

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_1 + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \beta_{ij} x_i x_j + e$$

where y is a dependent measure (i.e., response variable),
B_i terms are the estimated model coefficients for pure linear effects,
B_{ii} terms are the estimated model coefficients for pure quadratic effects,
B_{ij} terms are the estimated model coefficients for two-factor interactions,
x_i is the main effect of the ith factor,
$x_i x_j$ is the interaction between the ith and jth variables,
x_i^2 is the pure quadratic term for the ith factor, and
e is the error term or difference between actual and estimated response values.

Least squares regression estimates of the beta parameters specified in the second-order polynomial response surface is given by the expression.

$$I3 = (X'X)^{-1}X'Y$$

where	X	is an n x p design matrix for a particular fractional factorial design (see Appendix N), augmented so that column 1 is a column of 1s so that the intercept may be estimated;
	X’	is the p x n transpose of the X design matrix (augmented);
	$(X' X)^{-1}$	is the p x p inverse of the sum of squares and cross products matrix; and
	Y	is the n x 1 column vector of responses for each condition run; and
	X’Y	is the p x 1 column vector that is the matrix product of the two matrices involved.

As with the fractional factorial design, the central composite design data is analyzed with regression methods and the ANOVA to assess the statistical signficance of each of the terms of the fitted regression equation. The selection of levels of each variable to economically collect data to build the empirical second-order polynomial equation has been discussed by Williges (1980) and Williges and Williges (1992).

In order to build such a model, data are needed to solve the least squares regression equations. Box and Wilson (195 1, cited in Williges, 1981) developed an experimental methodology that determines optimal combinations of various quantitative factors to define the response surface. The design approach is to use a composite three separate parts: a) 2^k factorial or 2^{k-1} fractional factorial design portion; b) 2k additional points that outline a star pattern in the factor space; and c) 1 center point from which the entire composite of points radiates. For this reason, this is referred to as a central composite design. Generally, any second order, central composite design can be specified with a total of T points:

$$T=F+2k+1$$

where F = 2^k (or 2^{k-p})
k = the number of factors under investigation.
p = a number that represents the degree of fractionation in the fractional factorial design (1 = one-half fraction; 2 = one-fourth fraction; 3 = one-eighth fraction; 4= one-sixteenth fraction)

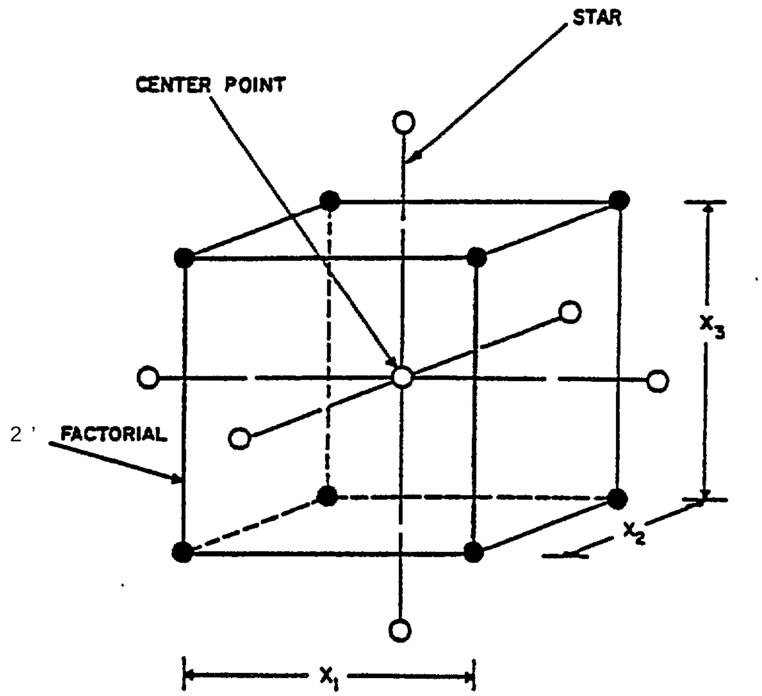
For example, consider a three-factor evaluation. If one substitutes k = 3 in the equation above, the value of T = 15, i.e., fifteen unique combinations are required to specify the second-order polynomial response surface. If one used a complete factorial experimental design, then with each factor at 5 levels (the number of levels needed to map out a second-order polynomial), there would be 5^3 or 125 combinations. The economy of approach is apparent.

Like the fractional factorial approach, this economy comes at a price. Care must be taken such that the 2^k combinations of factors, if substituted by a fractional factorial design portion due to the large number of factors of interest, be chosen so that all first- and second-order components are present and are not aliases of each other so that a complete second-order response surface can be generated (Williges, 1981). Clearly, all factors must be quantitative, else it makes little sense to discuss linear and quadratic components. While minimally only 3 levels of a factor are needed to outline a curve, 5 levels, appropriately chosen, will provide sufficient detail to develop an entire surface.

Figure 3-2 shows a hypothetical example of a three-factor central composite design to evaluate automobile driving performance (y) as a function of wind gust characteristics (Williges, 198 1). Only 15 unique treatment combinations are required, as indicated in the coding scheme at the bottom of the figure. Replication is needed to allow for the analysis of variance mean square error term to be defined. This may involve running two (or more) subjects in each of the treatment conditions or perhaps having a single group of subjects in a repeated measures format drive in all treatment conditions and thus provide replication over the entire design surface. See Williges (198 1) for more details about replication decisions.

The value of a remains to be specified. While there are alternative ways to define the a level (see Williges, 198 I), one simple way is such that the first and second-order beta weights are orthogonal (which facilitates least squares regression and analysis of variance:

DESIGN CONFIGURATION



Treatment Combination	X_1 Wind Gust Frequency	X_2 Wind Gust Velocity	X_3 Wind Gust Direction
1	+1	+1	+1
2	+1	-1	+1
3	+1	+1	-1
4	+1	-1	-1
5	-1	+1	+1
6	-1	-1	+1
7	-1	+1	-1
8	-1	-1	-1
9	+a	0	0
10	-a	0	0
11	0	+a	0
12	0	-a	0
13	0	0	+a
14	0	0	-a
15 .	0	0	.0

Figure F-2. Central Composite Design Illustration. Source: Williges (1980).

$$\alpha = \left(\frac{QF}{4}\right)^{1/4}$$

where Q = $[F + 2k + C)^{1/2} - F^{1/2}]^2$
C = the total number of center points (1 if equal replication is used)
F 2^k (or 2^{k-p})
k: the number of factors under investigation.
P= a number that represents the degree of fractionation in the fractional factorial design (1 = one-half fraction; 2 = one-fourth fraction; 3 = one-eighth traction; 4= one-sixteenth fraction)

For the three factor design, Q = $[(8 + 2(3) + 1)" - 8^{1/2}]^2$ = 1.092. This implies that a = $[(1.092)(8)/4]^{1/4}$ = 1.216. The application of the coding scheme is then applied by assigning the a codes to the lower and upper limits of the factor range of interest, the code 0 is assigned to the midpoint of the range, and the -1 and +l codes are assigned to factor values determined through linear interpolation. An example is provided in Table F-2. The design factors of interest are in-cab visual display luminance (x_1) (selectable over a range from 14 cd/m^2 to 140 cd/m^2), visual display contrast ratio (x_2) (selectable over a range from 2: 1 to 30:1), and symbol size (x_3) (selectable over a range from 10 to 28 arc-min). The response (y) is driver visual allocation time (number of glances x mean glance duration). The levels of each factor are included in Table F-2.

The central composite design, then, gets its name from the fact that it is a composite of a 2^k (or 2^{k-p}) design, augmented by a star pattern of data collection points that radiate from a center point. the Automated Experimental Design (AED) Assistant software (System Development Corporation, 1986) will also automatically generate central composite designs.

REPLICATION AND SINGLE-OBSERVATION FACTORIAL EXPERIMENTAL DESIGNS

In each of the preceding discussions, it was assumed that there were replications in each treatment combination of the experimental design in the form of two or more subjects in each treatment combination or having each subject perform in each treatment combination two or more times. In general, the greater the number of replicates in each treatment combination, the greater the precision in estimating error variance or experimental error. The price for this precision, however, is increased costs in data collection.

One simple approach to reduce the number of data observations to be collected is to eliminate replication altogether. This might be accomplished by assigning only one subject per treatment combination in a between-subjects experimental design. Alternatively, only one subject may be observed only once per treatment combination in a single-subject study. Many other schemes are also possible. Statistical tests of main effects and lower-order interactions are carried out by

Table F-2. Levels of three independent factors in Central Composite Experimental Design. (See text for explanation).

	Central Composite Design Codes and Associated Regressor Variable Values				
Regressor Variable	$-\alpha = -1.216$	-1	0	+1	$\alpha = 2.16$
x_1: Display Luminance	14 cd/m^2	23 cd/m^2	63 cd/m^2	127 cd/m^2	140 cd/m^2
x_2 : Contrast Ratio	2	4	14	27	30
x_3 : Character Size	10 arc-min	12 arc-mm	19 arc-min	26 arc-min	**28 arc-min**

Notes:

- See previous discussion for derivation of a.
- -a value is set to minimum of factor (regressor) range of interest
- +a value is set to maximum of factor (regressor) range of interest
- 0 value is set to mid-point of range from maximum to minimum of regressor range.
- -1 value is set to l/l .216 = .822 of the range between 0 and -a below the midpoint value
- +l value is set to l/l .216 = -822 of the range between 0 and -a above the midpoint value
- Note that all regressor values have been rounded.

pooling higher order interactions into a pooled-error term. This procedure assumes that the higher order terms are either non-significant or account for only a trivial proportion of the variability in the dependent measure. It is still possible that full factorial designs are infeasible because of the great number of treatment combinations. Under such conditions, the other economical data collection approaches should be considered.

SUMMARY

Efficient data collection supports efficient data analysis. For this reason, the data collection strategies presented here are beneficial to workload assessment and research. It is usually the case that the actual data collection and data capture vary somewhat from what was planned, especially in on-the-road evaluations or studies. The consultation of an experienced statistician or data analyst will be worthwhile to deal with the details of a statistical evaluation suitable for the data in hand.

REFERENCES

Box, G. E. P., Hunter, W. G., and Hunter, J. *S.* (1978). *Statistics for experimenters: An introduction to design, data analysis, and model building. New* York: Wiley.

Box, G. E. P., & Wilson, *K.* B. (1951). *On the experimental attainment of optimum conditions.* Journal of the Royal Statistical Society, Series B (Methodological), 13, l-45.

Keppel, G. (1991). *Design and analysis: A researcher 's handbook* Englewood Cliffs, NJ: Prentice Hall.

Kirk, R. E. *(1982). Experimental design: Procedures for the behavioral sciences* (Second edition), Belmont, CA: Brooks/Cole.

System Development Corporation (1986, December). *AED Volume II: The User-Assisted Automated Experimental (Test) design Program; Version* IV, Wright Patterson AFB, OH: Crew System Ergonomics Information Analysis Center (CSERIAS).

Tijerina, L., Kiger, S. M., Rockwell, T. H., & Tornow, C. E. (1995, September). *Final report: Workload assessment of in-cab text message system and cellular phone use by heavy vehicle drivers on the road* (Contract No. DTHN22-91-C-07003). Columbus, OH: Battelle.

Williges, R. C. (198 1). Development and use of research methodologies for complex system/simulation experimentation. In M. Moraal and *K. Kraiss* (Eds.), *Manned system design* (pp. 59-87). New York: Plenum.

Williges, R. C. (1984). Evaluating human-computer software interfaces. *Proceedings* **of** *the 1984 International Conference on Occupational Ergonomics,* Toronto, Canada, May, 1984.

Winer, B. J. (1971). *Statistical principles in experimental design* (2nd edition). New York: McGraw-Hill.

ATTACHMENT

EXAMPLE 2^K FRACTIONAL FACTORIAL DESIGNS

Design Matrix, 2^{3-1}, Resolution III Fractional Fractorial Design

Subject	Factor Level		
	1	2	3
1	-	-	-
2	+	-	+
3	-	+	+
4	+	+	-

Alternate Design Matrix:

Subject	Factor Level		
	1	2	3
1	-	-	+
2	+	-	-
3	-	+	-
4	+	+	+

Design Matrix, 2^{4-1}, Resolution IV Fractional Fractorial Design

Subject	Factor Level			
	1	2	3	4
1	-	-	-	-
2	+	-	-	+
3	-	+	-	+
4	+	+	-	-
5	-	-	+	+
6	+	-	+	-
7	-	+	+	-
8	+	+	+	+

Design Matrix, 2^{5-1}, Resolution V Fractional Fractorial Design

Subject	Factor Level				
	1	2	3	4	5
1	-	-	-	-	+
2	+	-	-	-	-
3	-	+	-	-	-
4	+	+	-	-	+
5	-	-	+	-	-
6	+	-	+	-	+
7	-	+	+	-	+
8	+	+	+	-	-
9	-	-	-	+	-
10	+	-	-	+	+
11	-	+	-	+	+
12	+	+	-	+	-
13	-	-	+	+	+
14	+	-	+	+	-
15	-	+	+	+	-
16	+	+	+	+	+

Design Matrix, 2^{6-1}, Resolution VI Fractional Fractorial Design

Subject	Factor Level					
	1	2	3	4	5	6
1	-	-	-	-	-	-
2	+	-	-	-	-	+
3	-	+	-	-	-	+
4	+	+	-	-	-	-
5	-	-	+	-	-	+
6	+	-	+	-	-	-
7	-	+	+	-	-	-
8	+	+	+	-	-	+
9	-	-	-	+	-	+
10	+	-	-	+	-	-
11	-	+	-	+	-	-
12	+	+	-	+	-	+
13	-	-	+	+	-	-
14	+	-	+	+	-	+
15	-	+	+	+	-	+
16	+	+	+	+	-	-

Design Matrix, 2^{6-1}, Resolution VI Fractional Factorial Design (Continued)

Subject	Factor Level					
	1	2	3	4	5	6
17	-	-	-	-	+	+
18	+	-	-	-	+	-
19	-	+	-	-	+	-
20	+	+	-	-	+	+
21	-	-	+	-	+	-
22	+	-	+	-	+	+
23	-	+	+	-	+	+
24	+	+	+	-	+	-
25	-	-	-	+	+	-
26	+	-	-	+	+	+
27	-	+	-	+	+	+
28	+	+	-	+	+	-
29	-	-	+	+	+	+
30	+	-	+	+	+	-
31	-	+	+	+	+	-
32	+	+	+	+	+	+

Design Matrix, 2^{7-1}, Resolution VII Fractional Factorial Design

Subject	Factor Level						
	1	2	3	4	5	6	7
1	-	-	-	-	-	-	+
2	+	-	-	-	-	-	-
3	-	+	-	-	-	-	-
4	+	+	-	-	-	-	+
5	-	-	+	-	-	-	-
6	+	-	+	-	-	-	+
7	-	+	+	-	-	-	+
8	+	+	+	-	-	-	-
9	-	-	-	+	-	-	-
10	+	-	-	+	-	-	+
11	-	+	-	+	-	-	+
12	+	+	-	+	-	-	-
13	-	-	+	+	-	-	+
14	+	-	+	+	-	-	-
15	-	+	+	+	-	-	-
16	+	+	+	+	-	-	+
17	-	-	-	-	+	-	-
18	+	-	-	-	+	-	+
19	-	+	-	-	+	-	+
20	+	+	-	-	+	-	-
21	-	-	+	-	+	-	+
22	+	-	+	-	+	-	-
23	-	+	+	-	+	-	-
24	+	+	+	-	+	-	+
25	-	-	-	+	+	-	+

Subject	Factor Level						
	1	2	3	4	5	6	7
26	+	-	-	+	+	-	-
27	-	+	-	+	+	-	-
28	+	+	-	+	+	-	+
29	-	-	+	+	+	-	-
30	+	-	+	+	+	-	+
31	-	+	+	+	+	-	+
32	+	+	+	+	+	-	-
33	-	-	-	-	-	+	-
34	+	-	-	-	-	+	+
35	-	+	-	-	-	+	+
36	+	+	-	-	-	+	-
37	-	-	+	-	-	+	+
38	+	-	+	-	-	+	-
39	-	+	+	-	-	+	-
40	+	+	+	-	-	+	+
41	-	-	-	+	-	+	+
42	+	-	-	+	-	+	-
43	-	+	-	+	-	+	-
44	+	+	-	+	-	+	+
45	-	-	+	+	-	+	-
46	+	-	+	+	-	+	+
47	-	+	+	+	-	+	+
48	+	+	+	+	-	+	-
49	-	-	-	-	+	+	+

Subject	Factor Level						
	1	2	3	4	5	6	7
50	+	-	-	-	+	+	-
51	-	+	-	-	+	+	-
52	+	+	-	-	+	+	+
53	-	-	+	-	+	+	-
54	+	-	+	-	+	+	+
55	-	+	+	-	+	+	+
56	+	+	+	-	+	+	-
57	-	-	-	+	+	+	-
58	+	-	-	+	+	+	+
59	-	+	-	+	+	+	+
60	+	+	-	+	+	+	-
61	-	-	+	+	+	+	+
62	+	-	+	+	+	+	-
63	-	+	+	+	+	+	-
64	+	+	+	+	+	+	+

Design Matrix, 2^{8-2}, Resolution V Fractional Fractorial Design

Subject	Factor Level							
	1	2	3	4	5	6	7	8
1	-	-	-	-	-	-	+	+
2	+	-	-	-	-	-	-	-
3	-	+	-	-	-	-	-	-
4	+	+	-	-	-	-	+	+
5	-	-	+	-	-	-	-	+
6	+	-	+	-	-	-	+	-
7	-	+	+	-	-	-	+	-
8	+	+	+	-	-	-	-	+
9	-	-	-	+	-	-	-	+
10	+	-	-	+	-	-	+	-
11	-	+	-	+	-	-	+	-
12	+	+	-	+	-	-	-	+
13	-	-	+	+	-	-	+	+
14	+	-	+	+	-	-	-	-
15	-	+	+	+	-	-	-	-
16	+	+	+	+	-	-	+	+
17	-	-	-	-	+	-	+	-
18	+	-	-	-	+	-	-	+
19	-	+	-	-	+	-	-	+
20	+	+	-	-	+	-	+	-
21	-	-	+	-	+	-	-	-
22	+	-	+	-	+	-	+	+
23	-	+	+	-	+	-	+	+
24	+	+	+	-	+	-	-	-

Design Matrix, 2^{8-2}, Resolution V Fractional Fractorial Design (Continued)

Subject	Factor Level							
	1	2	3	4	5	6	7	8
25	-	-	-	+	+	-	-	-
26	+	-	-	+	+	-	+	+
27	-	+	-	+	+	-	+	+
28	+	+	-	+	+	-	-	-
29	-	-	+	+	+	-	+	-
30	+	-	+	+	+	-	-	+
31	-	+	+	+	+	-	-	+
32	+	+	+	+	+	-	+	-
33	-	-	-	-	-	+	+	-
34	+	-	-	-	-	+	-	+
35	-	+	-	-	-	+	-	+
36	+	+	-	-	-	+	+	-
37	-	-	+	-	-	+	-	-
38	+	-	+	-	-	+	+	+
39	-	+	+	-	-	+	+	+
40	+	+	+	-	-	+	-	-
41	-	-	-	+	-	+	-	-
42	+	-	-	+	-	+	+	+
43	-	+	-	+	-	+	+	+
44	+	+	-	+	-	+	-	-
45	-	-	+	+	-	+	+	-
46	+	-	+	+	-	+	-	+
47	-	+	+	+	-	+	-	+
48	+	+	+	+	-	+	+	-
49	-	-	-	-	+	+	+	+

Design Matrix, 2^{8-2}, Resolution V Fractional Fractorial Design (Continued)

Subject	Factor Level							
	1	2	3	4	5	6	7	8
50	+	-	-	-	+	+	-	-
51	-	+	-	-	+	+	-	-
52	+	+	-	-	+	+	+	+
53	-	-	+	-	+	+	-	+
54	+	-	+	-	+	+	+	-
55	-	+	+	-	+	+	+	-
56	+	+	+	-	+	+	-	+
57	-	-	-	+	+	+	-	+
58	+	-	-	+	+	+	+	-
59	-	+	-	+	+	+	+	-
60	+	+	-	+	+	+	-	+
61	-	-	+	+	+	+	+	+
62	+	-	+	+	+	+	-	-
63	-	+	+	+	+	+	-	-
64	+	+	+	+	+	+	+	+